8 AREA & VOLUME

Areas of some well-known figures are given below:

Do You know ?

Square:

Diagonal = $a\sqrt{2}$

Rectangle:

Diagonal = $\sqrt{a^2 + b^2}$

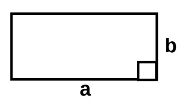
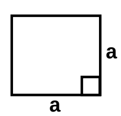
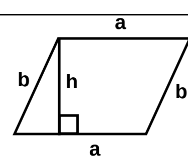
Parallelogram:

The diagonals bisect each other. Sum of adjacent angles = 180°

Rhombus:

The diagonals cut at right angles

$a^2 = (1/2d_1)^2 + (1/2d_2)^2$

S. No.	Name	Figure	Perimeter in units of length	Area in square units
1.	Rectangle	 a = length b = breadth	2(a + b)	ab
2.	Square	 a = side	4a	a^2 $\frac{1}{2}(\text{diagonal})^2$
3.	Parallelogram	 a = side b = side adjacent to a h = distance between the opp. parallel sides	2(a + b)	ah

TIPS

Quadrilateral:
In a cyclic quadrilateral sum of the opposite angles = 180°.

Trapezium:
The median is equal to half of the sum of the parallel sides.

Isosceles trapeziums have non-parallel sides equal.

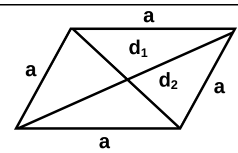
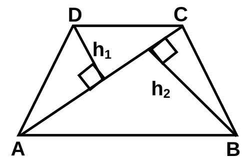
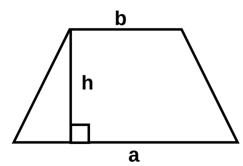
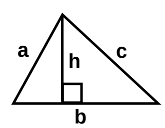
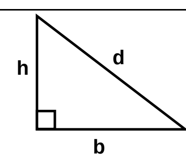
TIPS

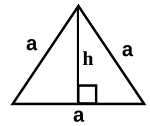
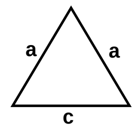
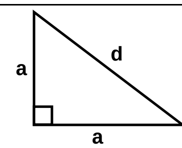
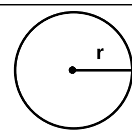
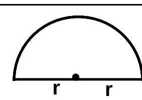
Right Angled Triangle
The mid point of the hypotenuse is the circum centre of the Δ circum radius = $\frac{1}{2}$ hypotenuse.

Equilateral Triangle:

r = $\frac{1}{3}$h

R = $\frac{2}{3}$h

4.	Rhombus	 a = side of rhombus; d_1,d_2 are the two diagonals	4a	$\frac{1}{2}d_1d_2$
5	Quadrilateral	 AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.	Sum of its four sides	$\frac{1}{2}$(AC) $(h_1 + h_2)$
6.	Trapezium	 a, b, are parallel sides and h is the distance between parallel sides	Sum of its four sides	$\frac{1}{2}h(a + b)$
7.	Triangle	 b is the base and h is the altitude. a, b, c are three sides of Δ.	a + b + c = 2s where s is the semi perimeter.	$\frac{1}{2}b \times h$ or $\sqrt{s(s-a)(s-b)(s-c)}$
8.	Right triangle	 d(hypotenuse) = $\sqrt{b^2 + h^2}$	b + h + d	$\frac{1}{2}bh$

9.	Equilateral triangle	 a = side h = altitude = $\frac{\sqrt{3}}{2}$ a	3a	(i) $\frac{1}{2}$ ah (ii) $\frac{\sqrt{3}}{4}a^2$
10.	Isosceles triangle	 c = unequal side a = equal side	2a + c	$\frac{c\sqrt{4a^2 - c^2}}{4}$
11.	Isosceles right triangle	 d(hypotenuse) = a$\sqrt{2}$ a = Each of equal sides. The angles are 90^o, 45^o, 45^o.	2a + d	$\frac{1}{2}a^2$
12.	Circle	 r = radius of the circle $\pi = \frac{22}{7}$ or 3.1416	$2\pi r$	πr^2
13.	Semicircle	 r = radius of the circle	$\pi r + 2r$	$\frac{1}{2}\pi r^2$

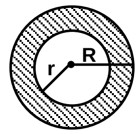
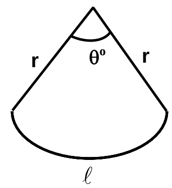
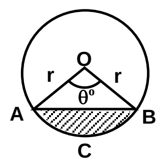
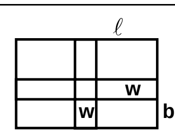
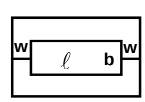
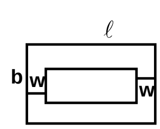
Do You know ?

(i) When a circle is inscribed in a square, then the side of the square = the diameter of the circle.

(ii) When a square is inscribed in a circle, the diagonal = the diameter of the circle.

(iii) A rectangle inscribed in a circle will have maximum area, if it is a square.

(iv) A triangle inscribed in a circle will have maximum area if it is an equilateral triangle.

14.	Ring (shaded region)	 R = outer radius r = inner radius	….	$\pi (R^2 - r^2)$
15.	Sector of a circle	 θ^o = central angle of the sector r = radius of the sector ℓ = length of the arc	$\ell + 2r$ where $\ell = \frac{\theta}{360} \times 2\pi r$	$\frac{\theta}{360} \times \pi r^2$
16.	Segment of a circle	 r = radius θ^o = angle of the related sector AOB	$\ell = \frac{\theta}{360} \times 2\pi r + 2r\sin\frac{\theta}{2}$	Area of segment ACB (Minor segment) $= r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$
17.	Pathways running across the middle of the rectangle	 ℓ = length b = breadth w = width of the path	$2(b + \ell)$	$A = w(\ell + b - w)$
18.	Pathways outside			$A = 2w(\ell + b + 2w)$
19.	Pathways inside			$A = 2w(\ell + b - 2w)$

If the area and perimeter of a rectangle are 240 cm^2 and 68 cm respectively, find its length and breadth.

Sol. Area A = 240 cm^2

Perimeter P = 68 cm

Length l =? & Breadth b =?

$A = \ell b = 240$ ………(1)

& $P = 2[\ell + b] = 68$

$\Rightarrow \ell + b = 34$ ………(2)

$\therefore$ From (1) & (2)

$\ell(34 - \ell) = 240$ or $\ell^2 - 34\ell + 240 = 0$

$(\ell - 24)(\ell - 10) = 0$

$\Rightarrow \ell = 24$ or $\ell = 10$

$\Rightarrow b = 10$ or $b = 24$

The two adjacent sides of a parallelogram are 12 and 14 metres respectively, and if the diagonal connecting the ends is 22 metres, find the area of the parallelogram.

Sol. One side a = 12 metres

Second side b = 14 metres

Diagonal c = 22 metres

In a parallelogram,

A = 2 Area of a triangle $\left\{\text{As the diagonal divides a ||gm into 2 parts of equal area}\right\}$

$$A = 2\sqrt{S(S-a)(S-b)(S-c)}$$

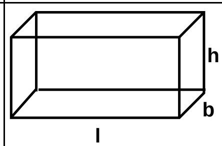
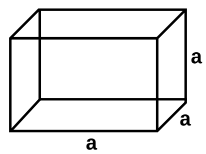
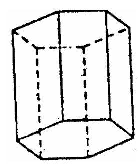
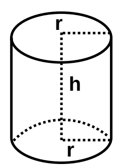
Where, $S = \dfrac{a+b+c}{2} = \dfrac{12+14+22}{2} = 24$

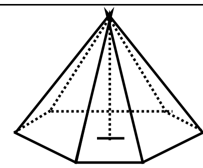
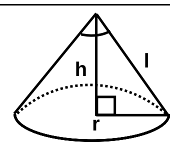
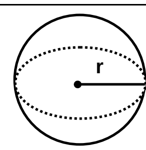
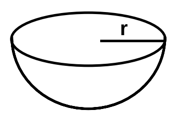
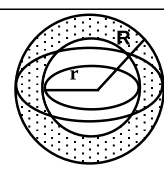
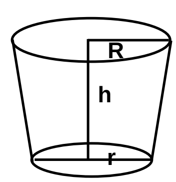
$$\therefore A = 2\sqrt{24 \times (24-12)(24-14)(24-22)}$$

$$= 2\sqrt{24 \times 12 \times 10 \times 2} = 24\sqrt{10} \times 2 = 151.78 \text{ m}^2.$$

VOLUME

> ***Do You know ?***
>
> **Cuboid:**
>
> The length of diagonal $= \sqrt{l^2 + b^2 + h^2}$
>
> **Cube:**
>
> The length of the diagonal $= a\sqrt{3}$

S. No	Nature of the solid	Shape of the solid	Lateral/ curved surface area	Total surface area	Volume
1.	Cuboid	 l = length	2h (l + b)	2(lb + bh + lh)	lbh
2.	Cube	 a = edge	$4a^2$	$6a^2$	a^3
3.	Right prism		(perimeter of base) × Height	2 (area of one end) + lateral surface area	Area of base × height
4.	Right circular cylinder	 r = radius of base h = height of the cylinder	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$

S. No	Nature of the solid	Shape of the solid	Lateral/ curved surface area	Total surface area	Volume
5.	Right pyramid		$\frac{1}{2}$ (Perimeter of the base) $\times$ (slant height)	Area of the base + lateral surface area	$\frac{1}{3}$ (Area of base) $\times$ height
6.	Right circular cone	 h = height r = radius l = slant height	$\pi r l$	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
7.	Sphere	 r = radius	—	$4\pi r^2$	$\frac{4}{3}\pi r^3$
8.	Hemi-sphere	 r = radius	$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\pi r^3\right)$
9.	Spheric -al shell	 R = outer radius r = inner radius	—	$4\pi (R^2 - r^2)$	$\frac{4}{3}\pi(R^3 - r^3)$
10.	Volume of bucket				$\frac{\pi h}{3}(R^2 + r^2 + Rr)$